CSCM12: software concepts and efficiency
Datastructures for ordered collections

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Algorithms and datastructures

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How stuff is layed out in memory matters

We will still discuss algorithms and efficiency

- Introducing datastructures → tools to
  - program efficiently **and/or**
  - representations for input/outputs for algorithmic problems

Today

- Some high-level considerations (not too long)
- Lists in python: dynamic arrays
- Another kind of lists: linked lists

Wot's a datastructure?

Informal concept, high-level

Rough reductionist definition

1. A chunk of memory space layed out in a specified way
   (the attributes an object of a class)
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Purpose?

Language-independent designation for a useful reusable abstraction

Examples: arrays (`int[]`), dynamic arrays (python lists, `ArrayList`), strings, sets

Interface and comparing datastructures

What is a good datastructure?

- Depends on the application/purpose
- Point of comparison: the operations

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Interfaces (again, informal)

The type signatures of operation and their **specification**

In the book: called **abstract datatypes**

Example: sets

You could imagine sets being any reasonable class that has the following methods:

```
class naiveSet:
  def __init__(self, c = []):
     self.elts = [i for i in c]
  def remove(self,e): # remove e
     self.elts = [i for i in self.elts, if i != e]
  def __contains__(self,e): # do I contain e the element?
     return e in self.elts
  def __add__(self,e); # add one element
     self.elts.append(e)
  def union(self, Set s); # adds all elements of ss
     self.elts += s.elts
```

Non-OO version of the same

For didactic purposes; more idiomatic in some programming languages

```
class naiveSet:
  def __init__(self, c = []):
     self.elts = [i for i in c]
def remove(s,e): # remove e from s
  s.elts = [i for i in s.elts, if i != e]
def contains(s,e): # do I contain e the element?
  return e in self.elts
def add(s,e); # add one element
  s.elts.append(e)
def union(s1, s2); # adds all elements of ss
  s1.elts += s2.elts
```

Comparing different implementations

Different valid **implementations** for a same inferface

- many parameters for comparison:

  time/space complexity, destructive or non-destructive update, ...

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Complexities for some implementations of `Set` (we will compute those later)

Op \Data	arrays	linked lists	Dynamic arrays	Sets (AVL)
`Set(T)`	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$
`remove`	$\mathcal{O}(n)$	$\mathcal{O}(n)$	$\mathcal{O}(n)$	$\mathcal{O}(\log(n))$
`contains`	$\mathcal{O}(n)$	$\mathcal{O}(n)$	$\mathcal{O}(n)$	$\mathcal{O}(\log(n))$
`add`	$\mathcal{O}(n)$	$\mathcal{O}(1)$	$\mathcal{O}(n)$ $\mathcal{O}(1)$ amortized	$\mathcal{O}(\log(n))$
`union`	$\mathcal{O}(n+m)$	$\mathcal{O}(1)$	$\mathcal{O}(n+m)$ $\mathcal{O}(m)$ amortized	$\mathcal{O}(m\log(n))$

Datastructure for collections

For today, we will be looking at datastructures for ordered collections

- I won't give a formal definition
- but essentially, the interface of lists

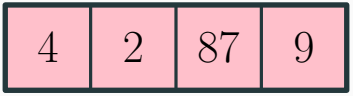
Typical operations

- Unique conversion to an array
- adding elements (arbitrarily or at a given indexed)
- removing by name/index.

Impementation of arrays (C, java, ...)

Arrays are just contiguous chunks in memory, represented by an address

(and an integer for the size in languages like java)



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Some properties

- reading a cell at a given index is constant-time

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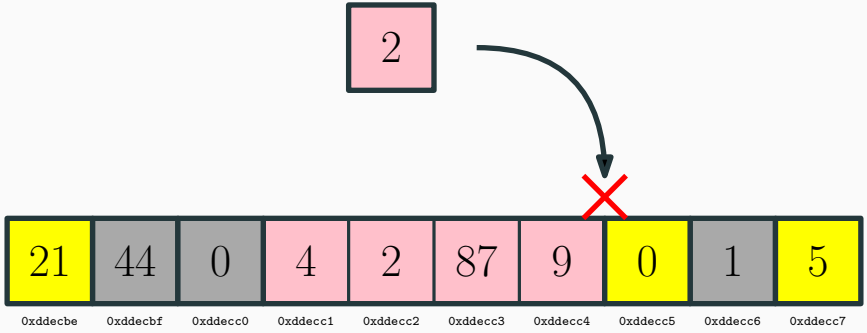
(this does not matter for *asymptotic* complexity)

Some things that are not too efficient with arrays

- Creating an array of size n is $\mathcal{O}(n)$
- **fixed size** $\Longrightarrow$ to extend, we need to re-create and copy an array
- Reason: your OS might have already allocated the memory around an array for some other purpose, so no guarantees of having anything free.



Can we do better?

Can we have $\mathcal{O}(1)$ insertion/deletion? At the end? Beginning? Middle?

Dynamic arrays

The answer is the workhorse behind `ArrayList<T>`

In a nutshell

An overlay on top of an array with a smart memory management policy.

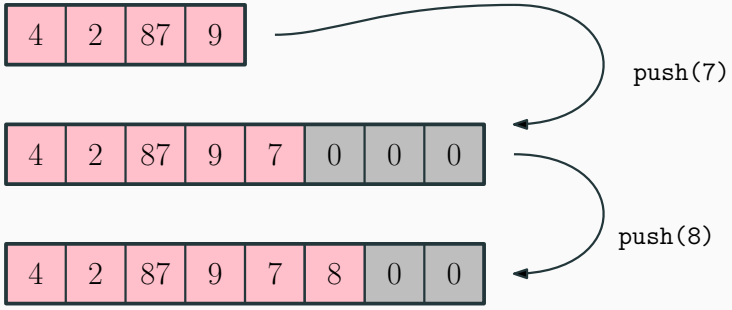
```
public class DynArrayInt {
private int[] internalArray;
private int size;
  ... }
```

Invariant: the size of internalArray is $= 2^{\lceil \log_2(\texttt{size}) \rceil}$

- This is more than needed
- Idea: plan ahead and reserve some space for future additions

Adding an element in a dynamic array

Let's picture adding 7 and 8 at the end of our running example:



Complexity: sometimes $\geq n$, sometimes $\mathcal{O}(1)\ldots$

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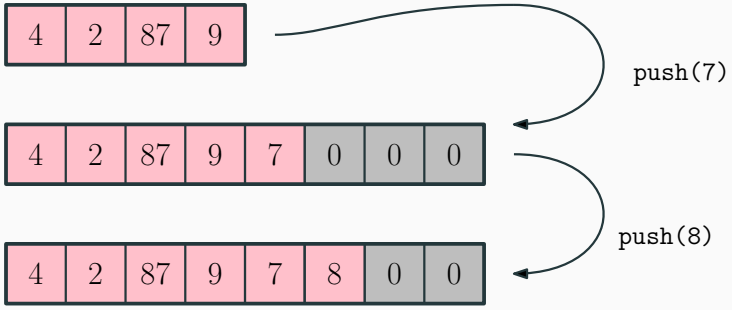

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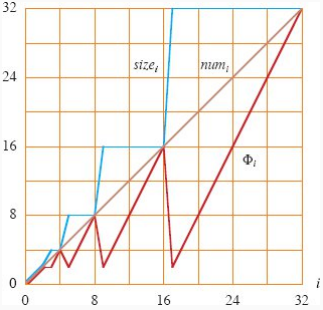
Constant amortized complexity!

Adding n elements to an *empty array* is $\mathcal{O}(n)$

Amortized complexity?

Idea: average during the whole lifecycle of a datastructure



(Cormen et. al., Intoduction to Algorithms 4th ed., fig 16.4)

Morally for python lists

- You can *mostly* pretend `append` is $\mathcal{O}(1)$
- But not always (e.g. real-time applications).
- (Slightly less space efficient than arrays)

But what if...

... we want $\mathcal{O}(1)$, even if we're starting from a big structure?

Simply linked lists: high-level idea

Recursive definition

A linked list is either

- a flag denoting an empty list
- or a cell containing a value and a reference to a linked list

Useful vocabulary for non-empty values

- **head** = value of the first cell
- **tail** = the remainder of the list

Example implementation in java

We need to use **recursively defined classes**

```
class llist:
  def __init__(self, x, n = None):
    self.head = x     # head = first value
    self.tail = n     # tail = rest of the list
                      #   (by default empty)
```

Slight issue: the empty list

- Can be simulated `None`
- But a better way would be to separate the recursive class from the global structure; what's above is kinda bad actually.

In practice

Still, let's use that for now (proper implementation: OOP exercise)

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```

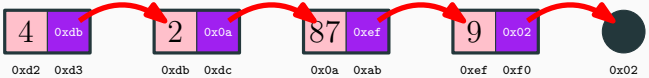
Model our example and get the third element:

```
tttail = llist(9);
ttail = llist(87,tttail);
tail = llist(2,ttail);
ex = llist(2,tail);
third = ex.tail.tail.head;
```

Quick comment about the memory layout

Not necessarily contiguous!

- Typically elements that are added in quick succession might be close, but up to implementation of the Python runtime



Adding an element in front: $\mathcal{O}(1)$!

Inserting an element (OO-style)

Suppose we want to insert an integer `x` at index `i`:

- Typically, recursion is nice to operate over recursively defined classes:

```
def insertAtIn(x, i, l):
  if i == 0:
    return llist(x, l)
  else:
    return llist(x, insertAtIn(x, i-1, l.tail))
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Complexity: $\mathcal{O}(i)$

The same with loops

Lists can also be rather easily handled with loops

```
def insertAtIn(x, i, l):
  if i == 0:
    return llist(x, l)
  u = l
  while i > 1:
    u = u.tail
    --i
  u.tail = llist(x,u.tail)
  return l
```

(can you spot the subtle difference between this implementation and the previous one?)

Exercises!

Setting an element at index i	$\mathcal{O}(i)$
Deleting an element at index i	$\mathcal{O}(i)$
Reversing a list of size n	$\mathcal{O}(n)$
Array conversion	$\mathcal{O}(n)$
Concatenating	

Exercises!

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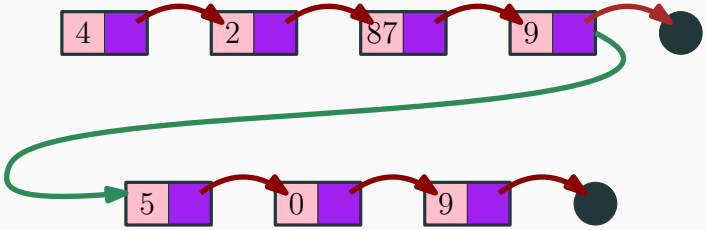
The issue with concatenation

It seems concatenation should be $\mathcal{O}(1)$

- Just modify the last tail pointer!



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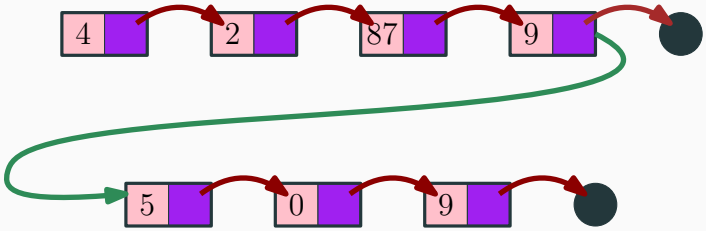


Solution: modify the datastructure to include a pointer to the end!

- To check: other operations doable with the same complexity

  that happens to be true here

- Similar exercise: adapt the datastructure so that reverse is $\mathcal{O}(1)$

  add a boolean to simulate reversing and adapt

Representing linked lists in OO properly

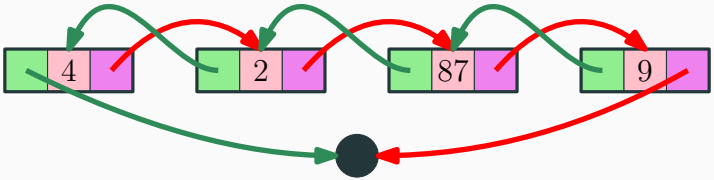
```
class cell:
  def __init__(self, v, n = None):
    self.head = v
    self.tail = n

class llist:
  def __init__(self, col = []): #coll is some iterable thing
    self.first = None # a cell
    self.last = None
    for i in col:
      self.append(i)
  # ... implementation
```

The recursion is still essential, but not in `llist`.

Further improvement: bidirectional links

Further improvement: doubly-linked lists



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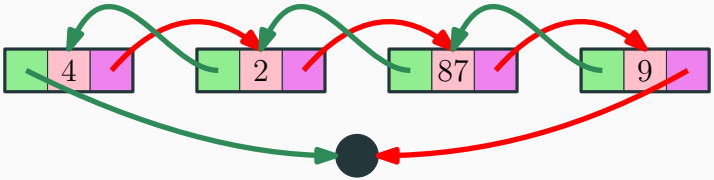
Further improvement: doubly-linked lists



- In practice, that is what Java does for `List<T>`
- easier to navigate around → insertion in $\mathcal{O}(\min(i, n-i))$
- hard to do doubly-linked lists with *non-destructive* updates

(straightforward for singly linked-list, hence why they are useful)

In java

```
class MyCell {
  MyCell prev;
  int head;
  MyCell next;
}

class MyDoublyLinkedList {
  protected MyCell start;
  protected MyCell last;
  ...
}
```

Lab task: filling in (some of) the rest! (but in python)

Comparison with arrays

Op \Data	`Array`	`List`
deletion/insertion at i	$\mathcal{O}(n)$	$\mathcal{O}(i)$
getting/replacing the value at i	$\mathcal{O}(1)$	$\mathcal{O}(i)$
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- Note that everything is linear time (rather fast in the grand scheme of things)
- For batch processing where we can bound the size statically → arrays win
- For **real-time** simulations (e.g. video games) with unbounded collections → lists win

To wrap up

Worth recalling the example comparison with the examples we have seen:

Complexities for some implementations of `Set`

Op \Data	`Array`	`List`	`ArrayList`
`{}`	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$
`remove`	$\mathcal{O}(n)$	$\mathcal{O}(n)$	$\mathcal{O}(n)$
`contains`	$\mathcal{O}(n)$	$\mathcal{O}(n)$	$\mathcal{O}(n)$
`add`	$\mathcal{O}(n)$	$\mathcal{O}(1)$	$\mathcal{O}(n)$ $\mathcal{O}(1)$ amortized
`union`	$\mathcal{O}(n+m)$	$\mathcal{O}(1)$	$\mathcal{O}(n+m)$ $\mathcal{O}(m)$ amortized

(Table limited to set operations while we have considered more operations in the lecture)

(e.g. insertion; dynamic arrays are not better than arrays at this)

Immutable datastructures

Datatypes that *cannot* be modified.

Python strings and tuples = **immutable** arrays

```
x = ('a', [1, 2])
x[0] = 2 # Error!
x[1][0] = 5 # No error (brainteaser for you!)
```

What's the point?

- can enable optimizations
- e.g. CPython caches short strings
- (nicer for concurrent settings)

Syntactic sugar and abstract datatypes

You can define your own custom classes to be compatible with the nice syntax for list comprehension and loops.

```
xs = myClassOfCollection(blabla)
print([j * k for j in xs, k in ks, if j != k])
```

You need to implement a special method

```
def __iter__(self):
```

which returns an object implementing "a traversal"

See `https://docs.python.org/3/reference/datamodel.html` §3.3.7 for details (and this weeks' lab!)

Other methods nice to overload

```
def __str__(self): # pretty-printing for the REPL

def __len__(self): # length
def __getitem__(self, index) # a[i]
def __setitem__(self, index, val) # a[i] = v
def __contains__(self, val) # membership

def __eq__(self, other): # x == y
def __le__(self, other): # x <= y

def __add__(self, other): # x + y

...
```

See `https://docs.python.org/3/reference/datamodel.html` for more

What comes next?

Next week, three abstract datatypes with applications:

- stacks
- queues
- priority queues

That will wrap up Chapter 4 of the book.

Then we will start on tree-like datastructures (chapter 7)