CSCM12: software concepts and efficiency
Minimalistic maths top-up

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Numbers

- Natural numbers $\mathbb{N} = \{0, 1, 2, \ldots\}$
- Integers $\mathbb{Z}$: includes negative numbers like -1
- Rationals $\mathbb{Q}$: fractions like $\frac{2}{3}$
- Real number $\mathbb{R}$: number that index *every point of a line*

(**very infinite very scary**)

Remember (for your real life)

`float`/`double` are only **approximations** of real numbers

- Finite representation in 32/64bits $\Rightarrow$ rounding
- Do **not** compare them for equality!

Bases

Natural numbers can be written in any base $b \in \mathbb{N} \setminus 0, 1$.

$$31 = \overline{31}^{10} = \overline{\mathtt{1f}}^{16} = \overline{11111}^{2}$$

Formally the expansion of a number n in base b is the unique sequence of $a_0, a_1, \ldots$ with $0 \leq a_i < b$ such that

$$n = \sum_i a_i b^i$$

Functions

A function $f : A \to B$ is relation mapping each a in A to a **unique** element of B which we write $f(x)$,

- A is the domain of f — its set of inputs
- B is the codomain of f — a set of potential outputs

WARNING

In CS, "function" sometimes has a different meaning: a procedure/algorithm/program

- Side-effects (modify memory, non-determinism, etc)
- In maths we are not interested in *how* $f(x)$ is computed
- In CS, we may care about running-time or other intensional aspects

Monomials

Functions $\mathbb{R} \to \mathbb{R}$ of the shape

$$x \longmapsto cx^n$$

Examples: x^2, $6x^5$

- Non-integer coefficients: e.g. $x^{\frac{1}{2}} = \sqrt{x}$
- NB: $x^{n+m} = x^n x^m$ and $(x^n)^m = x^{nm}$
- The bigger the exponent, the faster a monomial grows.

Polynomial functions

Functions $\mathbb{R} \to \mathbb{R}$ of the shape

$$x \longmapsto a_n x^n + \ldots + a_0 x^0$$

Examples:

- $x^2 + x + 5$
- $3x^{45} + 4x^7 + 2x$
- 5

- The degree of a polynomial is the largest n such that x^n occurs in its expansion.
- Rule of thumb: the higher the degree, the faster the polynomial grows

Exponential functions

Functions $\mathbb{R} \to \mathbb{R}_{>0}$ of the shape

$$x \longmapsto a^x$$

Examples: 2^x, 3^x, e^x

- Any exponential function ultimately grows **faster** than any polynomial

Logarithms

$\log_2$ (in base 2) is a function $\mathbb{R}_{>0} \to \mathbb{R}$

- For any integer $n \in \mathbb{N}$

$$\log_b(n) \approxeq \text{number of digits in } n \text{ in base } b$$

(more precisely: $\lfloor \log_b(n+1) \rfloor$)

- By convention (CS), $\log = \log_2$
- $\log_b(x) = \frac{\log_2(x)}{\log_2(b)}$, $\log_b(b^x) = b^{\log_b(x)} = x$
- log grows **slower** than any polynomial