CSCM12: software concepts and efficiency
Datastuctures for ordered collections

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Algorithms and datastructures

Starting from today we are going to talk about datastructures

- More complicated datatypes
- Designing more abstractions
    - interfaces
    - invariants

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- Designing more abstractions
  - interfaces
  - invariants

We will still discuss algorithms and efficiency

- Introducing datastructures → new tools to
  - program efficiently in any context (small/large scale, interactive/batch)
  - representations for input/outputs for algorithmic problems

Today

- Some high-level considerations (not too long)
- Our first example: linked lists
- If time allows: dynamic arrays, amortized complexity

Wot's a datastructure?

Informal concept, high-level

Rough reductionist definition

1. A chunk of memory space layed out in a specified way

   (in java, often the attributes an object of a class)

2. A bunch of **operations** (in java, the methods)

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Purpose?

Language-independent designation for a useful reusable abstraction

Examples: arrays (`int[]`), dynamic arrays (`ArrayList`), strings (`String`)

Interface and comparing datastructures

What is a good datastructure?

- Depends on the application/purpose
- Point of comparison: the operations

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Solution: compare across **interfaces**

Interfaces (again, informal)

The type signatures of operation and their **specification**

In Java: often using `interface`

Example: a fragment of the `Set` interface

- Define an interface for datastructure `Set`
- Represent a *set* of `T`s

(caveat: not quite the official java interface)

```
Set(); // creates an empty set
void remove(T e); // removes one element
boolean contains(T e); // do I contain the element?
void add(T e); // add one element
Set union(Set s2); // adds all elements of s2
...
```

Non-OO version of the same

For didactic purposes

```
Set(); // creates an empty set
static Set remove(Set s, T e); // returns s - {e}
static boolean contains(Set s, T e); // returns whether e is in s
static Set add(Set s, T e); // returns s unioned with {e}
static Set union(Set s1, Set s2); // returns s1 unioned with s2
...
```

But also: useful when considering **immutable data**

- Mutable = references, immutable = hard data like **`int`**...
- But for now, we will mostly consider **destructive update**

(i.e. inputs are consumed/modified)

(OO meshes well with this, not so well with immutable stuff (see all those **`static`**))

Comparing different implementations

Different valid **implementations** for a same inferface

- many parameters for comparison:

  time/space complexity, destructive or non-destructive update, ...

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Complexities for some implementations of `Set` **(we will compute those later)**

Op \Data	`Array`	`List`	`ArrayList`	`TreeSet`
`Set(T)`	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$
`remove`	$\mathcal{O}(n)$	$\mathcal{O}(n)$	$\mathcal{O}(n)$	$\mathcal{O}(\log(n))$
`contains`	$\mathcal{O}(n)$	$\mathcal{O}(n)$	$\mathcal{O}(n)$	$\mathcal{O}(\log(n))$
`add`	$\mathcal{O}(n)$	$\mathcal{O}(1)$	$\mathcal{O}(n)$ $\mathcal{O}(1)$ amortized	$\mathcal{O}(\log(n))$
`union`	$\mathcal{O}(n+m)$	$\mathcal{O}(1)$	$\mathcal{O}(n+m)$ $\mathcal{O}(m)$ amortized	$\mathcal{O}(m\log(n))$

Datastructure for collections

For today, we will be looking at datastructures for ordered collections

- I won't give a formal definition
- but essentially, we are going to look at array-like interfaces

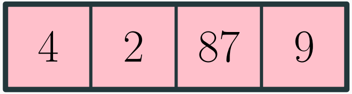
Typical operations

- Unique conversion to an array
- adding elements (arbitrarily or at a given indexed)
- removing by name/index.

Impementation of arrays

Arrays are contiguously represented in memory by an address

(and an integer for the size in languages like java)



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Source of the tradeoff for lists

Non-contiguous representation in memory, but still a linear stucture

Simply linked lists: high-level idea

Recursive definition

A linked list is either

- a flag denoting an empty list
- or a cell containing a value and a reference to a linked list

Useful vocabulary for non-empty values

- **head** = value of the first cell
- **tail** = the remainder of the list

Example implementation in java

We need to use **recursively defined classes**

```
class MyLinkedList
{
    int head;
    MyLinkedList tail;

    MyLinkedList(int nHead, MyLinkedList nTail) {
      head = nHead; tail = nTail;
    }
}
```

Slight issue: the flag for the empty list

- Can be simulated `null`
- But bad practice here for java

(cascade of design issues wrt encapsuation, ...)

In practice

Still, let's use that for the lecture (proper implementation: tedious OO exercise)

```
class MyLinkedList {
  int head;
  MyLinkedList tail;
}
```

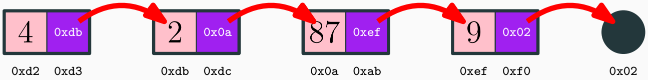
Model our example and get the third element:

```
MyLinkedList empty = null;
MyLinkedList tttail = new MyLinkedList(9,empty);
MyLinkedList ttail = new MyLinkedList(87,tttail);
MyLinkedList tail = new MyLinkedList(2,ttail);
MyLinkedList ex = new MyLinkedList(2,tail);
int third = ex.tail.tail.head;
```

Quick comment about the memory layout

Not necessarily contiguous!

- Typically elements that are added in quick succession might be close, but this is up to the implementation of **new**



Adding an element

The easiest thing is to add an element in front

- Non-OO-style:

```
static MyLinkedList push(MyLinkedList xs, int x){
  return new MyLinkedList(x, xs);
}
```

- OO-style:

```
MyLinkedList push(int x){
  return new MyLinkedList(x, this);
}
```

Careful: `xs.push(2)` does not modify `xs`

$\mathcal{O}(1)$!

Inserting an element (OO-style)

Suppose we want to insert an integer x at index i:

- Typically, recursion is nice to operate over recursively defined classes:

```
MyLinkedList insert(int i, int x){
  if(i == 0) then
    return push(x);
  tail.insert(i-1, x);
  return this;
}
```

Inserting an element (OO-style)

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```
MyLinkedList insert(int i, int x){
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```

Complexity: $\mathcal{O}(i)$

The same with loops

Lists can also be rather easily handled with loops

```
MyLinkedList insert(int i, int x){
  if(i == 0)
    return push(x);
  MyLinkedList previousNode;
  for(tmp = this; i > 1; --i)
     tmp = tmp.tail;
  tmp.tail = tmp.tail.push(x);
  return this;
}
```

Exercises!

Setting an element at index i	$\mathcal{O}(i)$
Deleting an element at index i	$\mathcal{O}(i)$
Reversing a list of size n	$\mathcal{O}(n)$
Array conversion	$\mathcal{O}(n)$
Concatenating	

Exercises!

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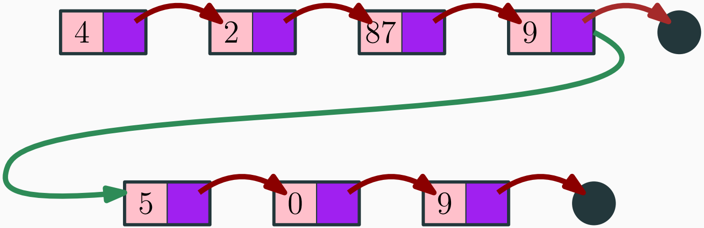
The issue with concatenation

It seems concatenation should be $\mathcal{O}(1)$

- Just modify the last tail pointer!



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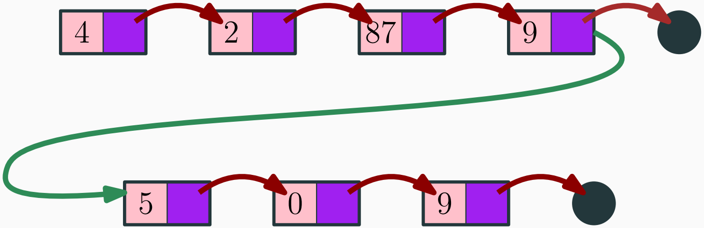


Solution: modify the datastructure to include a pointer to the end!

- To check: other operations doable with the same complexity

that happens to be true here

- Similar exercise: adapt the datastructure so that reverse is $\mathcal{O}(1)$

add a boolean to simulate reversing and adapt

Representing linked lists in OO properly

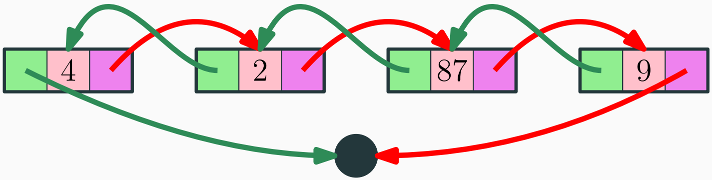
```
class MyCell {
  int head;
  MyCell tail;
}

class MyLinkedList {
  protected boolean empty;
  protected MyCell start;
  protected MyCell last;
  protected boolean flip;
  ...
}
```

The recursion is still essential, but not exposed by `MyLinkedList`.

Further improvement: bidirectional links

Further improvement: doubly-linked lists



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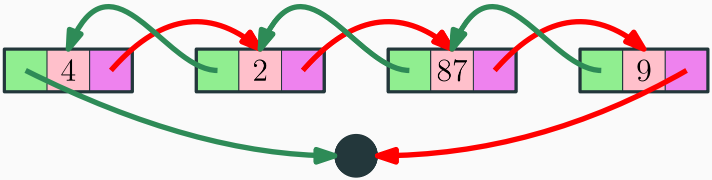

- In practice, that is what Java does for `List<T>`
- easier to navigate around $\rightarrow$ insertion in $\mathcal{O}(\min(i, n - i))$
- hard to do doubly-linked lists with *non-destructive* updates

(straightforward for singly linked-list, hence why they are useful)

In java

```java
class MyCell {
  MyCell prev;
  int head;
  MyCell next;
}

class MyDoublyLinkedList {
  protected boolean empty;
  protected MyCell start;
  protected MyCell last;
  protected boolean flip;
  ...
}
```

Comparison with arrays

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deletion/insertion at i	$\mathcal{O}(n)$	$\mathcal{O}(i)$
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Consequences:

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What about batch-processing with unbounded size?

Dynamic arrays

The answer is the workhorse behind `ArrayList<T>`

In a nutshell

An overlay on top of an array with a smart memory management policy.

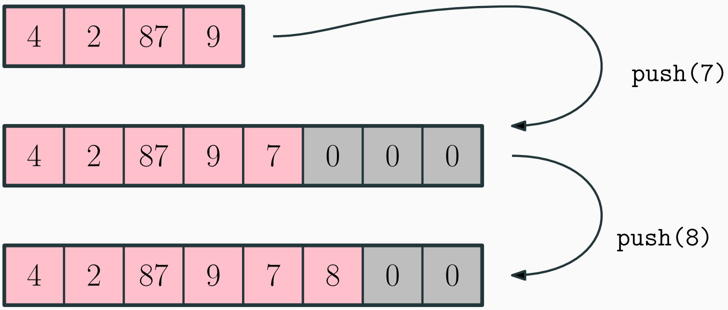
```
public class DynArrayInt {
private int[] internalArray;
private int size;
  ... }
```

Invariant: the size of internalArray is $= 2^{\lceil \log_2(\texttt{size}) \rceil}$

- This is more than needed
- Idea: plan ahead and reserve some space for future additions

Adding an element in a dynamic array

Let's picture adding 7 and 8 at the end of our running example:



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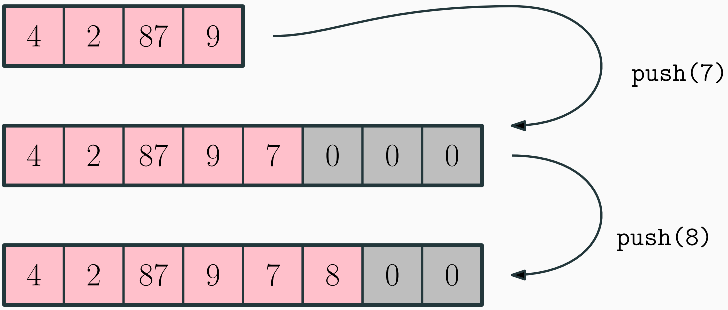

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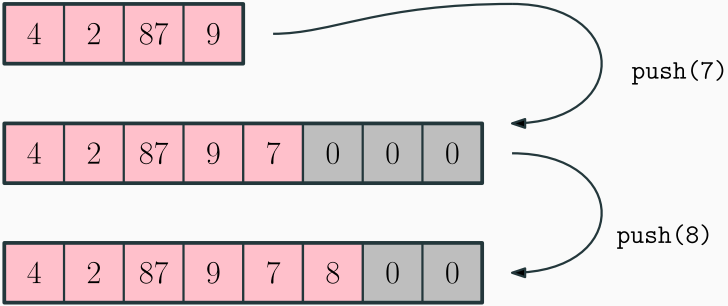

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Constant amortized complexity! **(advanced material, maybe later)**

Adding k elements to the empty array is $\mathcal{O}(k)$

To wrap up

Worth recalling the example comparison with the examples we have seen:

Complexities for some implementations of `Set`

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`Set(T)`	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$
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`add`	$\mathcal{O}(n)$	$\mathcal{O}(1)$	$\mathcal{O}(n)$ $\mathcal{O}(1)$ amortized
`union`	$\mathcal{O}(n+m)$	$\mathcal{O}(1)$	$\mathcal{O}(n+m)$ $\mathcal{O}(m)$ amortized

Note that this is limited to set operations while we have considered more operations in the lecture (e.g. insertion; synamic arrays are not better than arrays at this)