CSCM12 – Software concepts and efficiency March 6th 2023
Tonicha Crook & Cécilia Pradic

Coursework 1

1. Submission for this coursework will be individual; detailed submission instructions will be given on Canvas.

2. By submitting coursework, you state that you fully understand and are complying with the University's policy on Academic Integrity and Academic Misconduct. The policy can be found at `https://myuni.swansea.ac.uk/academic-life/academic-misconduct`. The consequences of committing academic misconduct can be extremely serious and may have a profound effect on your results and/or further progression. The penalties range from a written reprimand to cancellation of all of your marks and withdrawal from the University.

3. There is a total of 40 marks to be gained by answering the questions.

4. You get a 5 marks bonus for attending the labs – this will be checked by checking if you have done the first questions of the weekly labs up until March – but the total will still be over 40.

5. Please do this seriously and write things clearly – this is meant to be an opportunity to get ready for the exam and get some feedback on your individual written work before then. The other coursework will be purely coding.

Question 1: warm-up

For each of the following java function, give the asymptotic complexity in function of the input. Justify your answer. **[8 marks]**

1.
```java
static int[] fun1(int n)
{
  if(n == 0)
    return new int[2];
  int[] bla = fun1(n-1);
  bla[0] += 1 + bla[1];
  bla[1] *= 3;
  return bla;
}
```

2.
```java
static int fun2(int n)
{
  if(n <= 15)
    return 5;
  return fun2(n-1) * (1 - n/2);
}
```

3.
```java
static void fun3(int[][] arr)
{
  final int n = arr.length;
  if(n == 0 || arr[0].length != n)
    return;
  for(int k = 0; k < n; ++k)
  {
    if(k%2 == 0)
      for(int j = n; j > 0; --j)
        arr[k][j] = arr[j][k];
    else
      for(int j = 0; j < Math.sqrt(n); ++j)
        arr[j*j][k] = arr[k-1][j];
  }
}
```

4.
```java
static int fun4(int n)
{
  if(n <= 5)
    return 55;
  int r = 0;
  for(int i = 0; i < n; ++i)
    r = (n + 98 * r) % 23;
  return (fun4(n/3) + fun4(n/3 - 1) * r) % 55;
}
```

Question 2: problem

Call a user a *star* on a social media if they follow no one, but everyone else follows them. We want to find an algorithm such that, assuming that we are given as input a $n \times n$ matrix with `true` in cell (i, j) if user i follows user j and `false` otherwise, returns a user who is a star, or -1 if there is not any (by convention, let us say that users can't follow themselves so cells (i, i) can only contain `false`).

1. Give two examples of possible inputs with $n \geq 3$ users, one in which there is a star, and another where there is no star. **[2 marks]**
2. Is it ever possible to have two stars? Why? **[2 marks]**
3. Write an algorithm (in pseudo-code or in java, up to you) that solves the problem. The more efficient (asymptotically) your solution is, the higher the mark. **[5 marks]**
4. Run your algorithm step-by-step on the two examples you provided. **[2 marks]**
5. Analyze the asymptotic time complexity of your algorithm in function of the dimension of the input matrix. **[2 marks]**

Question 3: dual quicksort

The goal of this question is to investigate an implementation of a variant of quicksort used in some standard libraries where two pivots are used instead of one and the problem is reduced to three sub-problems instead of two.

1. Discuss the asymptotic time complexity of the quicksort algorithm in the best, average and worst case. Consider two variants of the pivot selection: one where the pivot is taken to be a random index in the array, the other where it taken to be the median of the collection of values in the array thanks to a linear-time algorithm. **[4 marks]**

   The Java implementation of our variant of the quicksort algorithm would be the following

```
static void dualQuickSortInner(int[] arr, min, max)
{
   if(max - min < 17)
     return insertSort(arr, min, max);

   putPivotsAtTheFrontAndBack(arr, min, max);
   int pivotsPos[] = pivot2(arr, min, max);
   dualQuickSortInner(arr, min, pivotsPos[0]);
   dualQuickSortInner(arr, pivotsPos[0]+1, pivotsPos[1]);
   dualQuickSortInner(arr, pivotsPos[1]+1, max);
}

static void dualQuickSort(int[] arr)
{
   dualQuickSortInner(arr, 0, arr.length - 1);
}
```

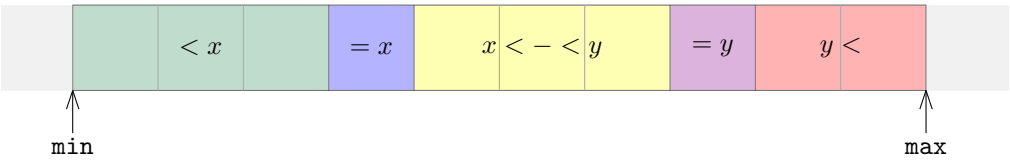
   To simplify the rest of the exercise, let us just assume that all the elements of `arr` are pairwise distincts. We will not specify the implementation of `insertSort` and `putPivotsAtTheFrontAndBack`, but will assume the following:

   - `putPivotsAtTheFrontAndBack(arr, min, max)` swaps up to 4 elements of the array between `min` and `max`, in such a way that `arr[min]` $\leq$ `arr[max-1]`. The informal idea is that it puts two special pivots elements at the front and at the back of the array
   - `insertSort(arr,min,max)` is a naive sorting algorithm that permutes the elements of `arr` between `min` and `max` so that they end up sorted
   - `pivot2(arr, min, max)` should permute elements between `min` and `max` and returns an array `pivots` with two positions

     $$\texttt{min} \leq \texttt{pivotsPos[0]} < \texttt{pivotPos[1]} \leq \texttt{max}$$

     such that, assuming we originally had $x =$ `arr[min]` and $y =$ `arr[max]` with $x < y$
     - between `min` and `pivotsPos[0]` (inclusive), all elements of `arr` are strictly smaller than x
     - from `pivotsPos[0]+1` to `pivotsPos[1]`, all elements of `arr` are in $]x, y[$
     - from `pivotsPos[1]+1` to `max`, all elements of `arr` are equal to y

Below is a picture synthesizing how the requirements above on the output between `min` and `max` will look like:



2. Write a java implementation or pseudo-code for

```
static int[] pivot2(int[] arr, min, max);
```

as specified above. Your solution should work in-place (i.e., without creating a new array) and in time $\mathcal{O}(\texttt{max} - \texttt{min})$. **[5 marks]**

3. Show that on input arrays that are already sorted, what is the time complexity of this sorting algorithm, assuming that `putPivotsAtTheFrontAndBack` does nothing? Justify briefly your answer by exhibiting the shape of a recurrence relation satisfied by this function. **[3 marks]**

4. Now assume that the function `putPivotsAtTheFrontAndBack` is smarter and manages to swap the elements so that the recursive calls are across three sub-arrays of roughly (up to ± 1) the same size. Give the recursive equation satisfied by the time complexity then. and deduce from the Master theorem the time complexity of `dualQuickSort`.

[5 marks]

5. Is it clear whether there is a superior solution between quicksort and that one through the asymptotic analyses carried out in previous questions? If not, suggest some ways to determine which is the better candidate.

[2 marks]