CSCM12 – Software concepts and efficiency March 24th 2023
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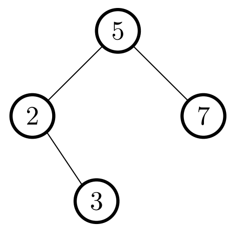
Lab 6: binary trees

For the lab this week, your task will be to complete the file `BTreeExercises.java`. Use the main function to test your functions and show your exampes if you want to sign off.

1. **Some examples**

   (a) In the `main` function, two examples `example1` and `example2` are defined. Draw the corresponding structures. Do these structures have sharing? Cycles?

   (b) Write some code that represent the following tree in a variable `example3`:

   (c) Run the `main` function and try to understand the output of `toString`. Try to define a variable `example4` such that `example4.ofString()` is `"(2 (4)) 5 (2)"`.

   (d) Define a further `example5` with a cycle. For testing, don't try to call `toString`; but it should be the case that `example5.hasCycle()` is `true`, so you can test that.

2. **Basic methods** In this question we look at starting to flesh out the class `BTree`

   (a) Write the body of the method

   ```
   public int depth()
   ```

   that returns the depth/height of the tree, i.e. the longest path from the root of the tree to a leaf.

   (b) Write the body of the method

   ```
   public boolean equals(BTree<T> otherTree)
   ```

   that checks wether the calling object represents the same tree as `otherTree` (it should *not* be the same as ==; think of what you know about `String`)

   (c) Write the body of the method

   ```
   public ArrayList<T> prefixTraversal()
   ```

that converts a list to an array using a prefix traversal.

(d) Write the body of the method

```
static public <T> BTree<T> of(ArrayList<T> arr)
```

that converts an array to a tree using the scheme presented in the lecture/the scheme in the last lab sheet presented in the last lab sheet to represents full binary trees using arrays (a `null` value in the array indicates the absence of any subtree).

3. **Binary search trees** Now we are going to look at methods of the class `BST`. Objects of this class have a single tree attribute, which should always be a binary search tree and all operations should preserve that invariant: for every node, the label should be greater than all labels on the left subtree of the node and lesser than the labels in the right subtree if they exist

   (a) Write the body of the method

   ```
   public void insert(int k)
   ```

   that inserts a new number in the binary search tree.

   (b) Write the body of the method

   ```
   public void delete(int k)
   ```

   that deletes a number from the binary search tree.

4. **Challenge: more advanced functions** Let us look at some more functions of the class `BTree`

   (a) Write the body of the method

   ```
   public <U> BTree<U> map(Function<T, U> f)
   ```

   that returns the tree obtained by relabelling uniformly all the nodes of the calling tree using `f` (recall you can use `f.apply(bla)` to apply `f`).

   (b) Write the body of the method

   ```
   static public BTree<Integer> of(String st)
   ```

   that should convert a `String` to a tree; this should be the inverse of the function `toString` that is provided

   (c) Write the body of the method

   ```
   public void unshareSubtrees()
   ```

   that removes sharing from a structure with no cycles, but keeps the same underlying unfolding (so the output of the provided `toString` method should remain the same)

   (d) Change the code of the `toString` function so that it prints out useful informations in case there are cycles. You could for instance write the result as some sort of system of recursive equations, as

$$(x)\ 5\ ((2)\ 4\ (x)) \qquad \text{where} \left\{ \begin{array}{lcl} x & = & ((5)\ x)\ 7\ y \\ y & = & y\ 4\ (x\ 5\ y) \end{array} \right.$$